1: Numerical Methods – 1: (8 hours): Solution of polynomial and transcendental equations – Bisection method, Newton-Raphson and Regula-Falsi method. Finite differences, Relation between operators, Interpolation using Newton's forward and backward difference e. Interpolation with unequal intervals: Newton's divided difference and Lagrange's formulae.

Algebraic and Transcendental equation: An equation of the form $f(x) = a_0x^n + a^1x^{n-1} + ... + an = 0$ is called an algebraic equation. An equation consisting trigonometric function, exponential function, logarithmic function etc. is called a transcendental equation.

Solution of Algebraic and Transcendental equations:

Bisection Method:

(i) Find the negative and positive values of the function at two different points ,
(ii) Say $f(a)$= - *Ve* and $f(b)$= +*ve* (Then Root lies b/w a and b)
(iii) Take $a=x_0$ and $b=x_1$
(iv) Find $x_2 = x_0 + x_1 /2$
(v) Find $f(x_2)$
(vi) If $f(x_2)$=+ve then root lies b/w $a=x_0$ and x_2
If $f(x_2)$= - ve then root lies b/w $b=x_1$ and x_2 , repeat procedure from (iii)

Q1. Find the root of the equation x^3- x- 4 = 0 which lies between 1 and 2 by **Bisection Method.** ***[ANS: $X_{10}=X_{11}$=1.79638]***

Q2. Find a real root of the equation $x \log_{10} x$ = 1.2 by **Bisection Method**. ***[ANS: $X_{12}= X_{13}$=2.714] [MAY19]***

Q3. Find the root of the equation x^3+x- 1 = 0 by Bisection Method , near 1 (up to 3 decimal places) **[May 18]**

Q4. Find the root of the equation x^3-4 x- 9 = 0 by **Bisection Method** correct to three decimal places**. [June 16,Nov. 18 ME/CE]**

Regula Falsi Method (Or Method of false positions)

1. find the negative and positive values of the function at two different points
2. say $f(a)$= -Ve and $f(b)$= +ve(Then Root lies b/w *a* and *b*)
3. let $a=x_0$ and $b=x_1$
4. Find $x_2 = \dfrac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$
5. find $f(x_2)$
6. If $f(x_2)$=+ve then root lies b/w $a=x_0$ and x_2
7. If $f(x_2)$= - ve then root lies b/w $b=x_1$ and x_2 , repeat procedure from (2)

Q5. Find the root of the equation x^3- 2x- 5 = 0 which lies between 2 and 3 by method of **false position.** ***[ANS: $X_6=X_7$=2.094] [JUNE 2010 , 17]***

Q6. Find the root of the equation x^3- 5x- 7 = 0 which lies between 2 and 3 by method of **false position**. ***ANS: $X_3=X_4$=2.7472][June2005 , may 18]***

Q7. Find the root of the equation x^3- 4x+1 = 0 by the method of **false position**. ***[ANS: X=0.25] RGPV JUNE 2007]***

Q8. Find the root of the equation $x^3+x^2 -3X-3 = 0$ by the method of **false position**. ***[ANS: X_3=1.728] [RGPV JUNE 2005]***

Q9. Find a real root of the equation $x \log_{10} x$ = 1.2 by **Regula-falsi method** . ***[ANS: X_3=2.74065][RGPV Dec. 07, JUNE 2009,June 16]***

Q10. Find a real root of the equation $2X-\log_{10}X=7$ by **Regula-falsi method** . ***[ANS: X_3=3.78928] RGPV JUNE 2005]***

Newton Rap son's Method:

1. Find the negative and positive values of the function at two different points
2. Say $f(a)$= -Ve and $f(b)$= +ve
3. If | $f(a$ |< | $f(b)$| **(Numerical Value, without sign) ,** then take $a= x_0$
4. Find $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$, Provided $f'(x_n)$ exist
5. Find net approximations using (2)

Q11. Find the real root of the equation x^4- x- 9 = 0 by **Newton-Raphson** method. ***[ANS: X_4=1.8134][RGPV JUNE 2006,June 16]***

Q12. Find the real root of the equation x^3- 3x+1 = 0 by **Newton-Raphson** method. ***[ANS: X_3=0.3473][RGPVDEC. 2009,2007]***

Q13. Find the real root of the equation x^3- 2x-5 = 0 by **Newton-Raphson** method. ***[ANS: X_4=2.09456][RGPV JUNE 2007]***

Q14. Find the real root of the equation 3*x*= cos *x*-1 by **Newton-Raphson** method.
[ANS: X_2=0.6071] [JUNE 2004,DEC.2006, Dec. 2002 June16may18 Nov. 18]

Q15. Find the real root of the equation x^4- x- 10 = 0 by Newton-Raphson method. [ANS: X4=1.85558][JUNE 08,DEC.2003]

Q16. Find a real root of the equation $x \log_{10} x$ 1.2 by **Newton-Raphson method** .

Q17. Find a real root of the equation $x \log_{10} x$ 1.2 by **Newton-Raphson method** ***[ANS: X_4=2.74065][RGPV FEB. 2010,DEC. 05]***

Q.17. Find a real root of the equation $X \log_{10}X=4.77$ by **Newton-Raphson method** .

Q18. Find the real root of the equation $x=e^{-x}$ by **Newton-Raphson** method. ***[RGPV may 19]***

Q19. Solve the algebraic equation $x^3+2x^2+10x-20=0$ by **Newton-Raphson** method. ***[RGPV may 19 Ex.]***

Secant Method: This method is same as the *Regula Falsi Method, but in this method* ***we not need to check the +ve and –ve sign in each step***. We can use general formula $x_{n+2} = \dfrac{x_n f(x_{n+1}) - x_{n+1} f(x_n)}{f(x_{n+1}) - f(x_n)}$

Q20. Find the root of the equation $x^3 - 2x - 5 = 0$ which lies between 2 and 3 by method of **Secant .** ***[ANS: $X_6=X_7=2.094$] [RGPV JUNE 17]***

Q21. Find a root of $\cos x - xe^x = 0$ by using Secant or Chord Method. ` **Ans: $x_6=x_7=0.5177$** **[Dec.12, Nov 2018]**

Difference operators:

1. Shifting Operator: $E f(x) = f(x+h)$, $E^2 f(x) = f(x+2h)$, ….. $E^n f(x) = f(x+nh)$, or $E y_x = y_{x+h}$, $E^n y_x = y_{x+nh}$,
2. *Forward difference operator:* $\Delta f(x) = f(x+h) - f(x)$ or $\Delta y_x = y_{x+h} - y_x$
3. *Backward difference operator :* $\nabla f(x) = f(x) - f(x-h)$ or $\nabla y_x = y_x - y_{x-h}$
4. *Averaging operator :* $\mu f(x) = \dfrac{f(x+\frac{h}{2}) + f(x-\frac{h}{2})}{2}$
5. *Central difference operator* $= \delta f(x) = f(x+\frac{h}{2}) - f(x-\frac{h}{2})$
6. $E = e^{hD}$ [Hint : use Taylor Series $f(x+h) = f(x) + h f'(x) + h^2/2 f''(x)$……. Then we get $Ef(x) = e^{hD} f(x)$]

$\Delta = E - 1$

$\nabla = 1 - E^{-1}$

$\mu = (E^{1/2} + E^{-1/2})/2$

$\delta = E^{1/2} - E^{-1/2}$

Q.1 Prove (i) $\Delta^2(3e^x) = 3(e^h - 1)^2 e^x$ (ii) $\Delta^2(\cos 2x) = -4\sin^2 h \cos(2x+2h)$

(iii) $\Delta(e^{ax}.\log bx) = e^{ax}\left[e^{ah}\log(\frac{x+h}{x}) + (e^{ah}-1)\log bx\right]$ (iv) $\Delta \tan^{-1} x = \tan^{-1}\left[\dfrac{h}{1+hx+x^2}\right]$

Q.2 Prove (i) $hD = -\log(1-\nabla)$ (ii) $hD = \log(1+\Delta)$ (iii) $hD = \sin h^{-1}(\mu\delta)$

Q.3 Prove that (1) $e^x = (\dfrac{\Delta^2}{E})e^x . \dfrac{Ee^x}{\Delta^2 e^x}$ **[Dec. 2015,June 2014, Feb. 2010, June 2009, 2008, 2007, Dec. 2006, 2004, June 2002]**

Q.4 Prove (i) $\Delta = \dfrac{\delta^2}{2} + \delta\sqrt{1+\dfrac{\delta^2}{4}}$ (ii) $\mu\delta = \dfrac{1}{2}(\Delta + \nabla)$ (iii) $(1+\nabla)(1+\Delta)=1$ (iv) $E\nabla = \nabla E = \Delta$ (v) $\nabla = \delta E^{-1/2}$ (vi) $\delta^2 = \nabla\Delta = \Delta - \nabla$

Q.5 Prove that $(E^{1/2} + E^{-1/2})(1+\Delta)^{1/2} = 2 + \Delta$ **[RGPV: June 2009, Dec. 2005]**

Q.6 Prove that $hD = \log(1+\Delta) = -\log(1-\nabla) = \sinh^{-1}(\mu\delta)$ **[RGPV: Dec. 2005]**

Find Missing Terms: If there are n missing terms/ data in the given table then $\Delta^{n-1} y_x = 0$ or $\Delta^{n-1} f(x) = 0$, use $\Delta = E - 1$and and expand the series using binomial theorem $(a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 + {}^nC_3 a^{n-3} b^3 + ... + {}^nC_n a^{n-n} b^n$

Or $(a+b)^n = a^n + na^{n-1}b^1 + \dfrac{n(n-1)}{2} a^{n-2} b^2 + \dfrac{n(n-1)(n-2)}{3} a^{n-3} b^3 + ... + b^n$

i.e $(E-1)^5 y_x = \left(E^5 + 5E^4(-1)^1 + \dfrac{5.(5-1)}{2}E^3(-1)^2 + \dfrac{5.(5-1)(5-2)}{3}E^2(-1)^3 + \dfrac{5.(5-1)(5-2)(5-3)}{4}E^1(-1)^4 + \dfrac{5.(5-1)(5-2)(5-3)(5-4)}{5}E^0(-1)^5\right) y_x = 0$

or $y_{x+5} - 5y_{x+4} + 10y_{x+3} - 20y_{x+2} + 10y_{x+1} - y_x = 0$ (Since $E^n y_x = y_{x+n}$), Put x= 0,1..... and solve the algebraic eq.s

Q.7 Find the missing term: **Ans:** 31 , **[RGPV: Dec. 2002]**

$x:$	0	1	2	3	4
$f(x):$	1	3	9	?	81

Q.8 Find the first term of the series whose second and subsequent terms are 8,3,0,-1,0.**Ans:15** **[Dec. 2007,June 2003]**

Q.9 Find the missing terms: **Ans:** f(60)=0.225,f(50)=2.925 **[RGPV: June 2007]**

$x:$	45	50	55	60	65
$f(x):$	3	?	2	?	−2.4

Q.10 Find the missing terms: **Ans:** f(10)=13.25,f(20)=22.5 **[RGPV: June 2006, June 2015]**

$x:$	0	5	10	15	20	25
$f(x):$	6	10	?	17	?	31

Q.11 Find the missing terms:

x:	2	2.1	2.2	2.3	2.4	2.5	2.6
$f(x)$:	0.135	–	0.111	0.100	–	0.082	0.074

Ans: f(2.1)=0.123,f(2.4)=0.0904 **[June 2004]**

Q.12 Assuming the following values of y belong to the polynomial of degree 4 & compute the next three values.

x:	0	1	2	3	4	5	6	7
y:	1	–1	1	–1	1	?	?	?

Ans: 31,129,351 [RGPV. Dec. 2004]

Q.13 Find the first term of the series whose second and sub sequent terms are 8,3,0,-1,0.

Factorial Polynomials: The factorial polynomial is the continued product of the factors in which the first factor is *x* and the successive factors decrease by a constant h and is denoted by $x^{(n)}$. Where $x^{(n)}=x(x-h)(x-2h)\ldots\ldots\ldots\{x-(n-1)h\}$

i.e. $x^{(1)}=x$, $x^{(2)}=x(x-1)$, $x^{(3)}=x(x-1)(x-2)\ldots\ldots$

$$\Delta x^{(n)}=nx^{(n-1)},\ \Delta^2 x^{(n)}=n(n-1)x^{(n-2)}\ \ldots\ldots \text{ and } \frac{1}{\Delta}x^{(n)}=\frac{x^{(n+1)}}{n+1},\ \frac{1}{\Delta^2}x^{(n)}=\iint x^{(n)}dx=\frac{x^{(n+2)}}{(n+1)(n+2)}$$

Q.14 Express the following function $2x^3-3x^2+3x-10$ in in a factorial Notations and hence show that $\Delta^3 y=12$.

Ans: $f(x)=2x^{(3)}+5x^{(2)}+2x^{(1)}-10$ **[RGPV: June 2007, Dec. 2006,June16]**

Solution :use synthetic subdivision method :

0	**2**	**-3**	**3**	**-10**
add	**0**	**0 *2=0**	**-3*0=0**	**3*0=0**
	2	**-3**	**3**	**-10**
1	**0**	**1*2=2**	**-1*1=-1**	
2	**2**	**-1**	**2**	
	0	**3*2=6**		
3	**2**	**5**		
	0			
	2			

Hence polynomial will be
: $f(x)=2x^{(3)}+5x^{(2)}+2x^{(1)}-10$

Q.15 Express $y=x^4-12x^3+24x^2-30x+9$ in a factorial and hence show that $\Delta^4 y=24$

Ans: $f(x)=x^{(4)}-6x^{(3)}-5x^{(2)}-17x+9$ **[RGPV: June 2004]**

Obtain the function whose first difference is $2x^3+3x^2-5x+4$ **Ans:** $½\left(x^4-7x^2+14x+C\right)$ **[Feb. 2010, Dec. 2007]**

Q.16 Obtain the function whose first difference is $9x^2+11x+5$ **Ans:** $3x^3+x^2+x+d$ **[RGPV: Dec. 2004, June 2015]**

INTERPOLATION:

Interpolation is the process to find the values of y for any intermediate value of x between the interval.
Extrapolation is the process to find the values of y for any value of x outside the interval.

Interpolation with equal intervals:

1. Gregory- Newton's Forward difference interpolation formula: When required value of y=f(x) is near to the top then use forward difference interpolation formulae.

$$y=f(x)=y_0+\frac{p}{1!}\Delta y_0+\frac{p(p-1)}{2!}\Delta^2 y_0+\ldots\ldots+\frac{p(p-1)\ldots\ldots(p-(p-1))}{n!}\Delta^n y_0\ldots\ldots\ldots\ldots+\left[Where\ p=\frac{(x-x_0)}{h}\right]$$

Q.17 Find the number of men getting wage Rs 10 from the following:

Wages in Rs.	5	15	25	35	**Ans:**
No. of Men	9	30	35	42	**22.625**

Q.18 Prepare the difference table for the following **[June 2015]**

x	10	20	30	40	50
y	12	15	20	27	39

Q.19 Estimate from the following table the number of students who obtained marks between 40 and 45: **[June 2001, June 2015]**

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No. of Students	31	42	51	35	31

[*Hint: Convert the table in below form and add frequencies to get cumulative frequencies (y)* .**Ans:48 nearly]**

Q.20 Find the cubic polynomial which takes the following value , a Hence or otherwise evaluate y(4)

x: 0 1 2 3

$f(x)$: 1 2 1 10

Ans: $2x^3-7x^2+6x+1$, **y(4)=4 ,** **[RGPV: Feb. 10 ,June 2010, Dec. 2014]**

Q.21 Use Newton's forward interpolation formula , find the cubic polynomial and hence evaluate f(0.5) by the following data:

x	0	1	2	3	4	**Ans:**
y	-1	0	13	50	123	**-1.25**

Q.22 The following table gives the population of a town during the last six censuses. Estimate using any suitable interpolation formula the increase in the population during the period from 1946 to 1948.

x: 1911 1921 1931 1941 1951 1961

$f(x)$: 12 15 20 27 39 52

Ans: ***f(1946)=32.3437, f(1948)=34.873215,*** **Difference=2.53 [June 2003]**

Q.23 The following table gives the velocity *v* of a particle at time *t* , find the distance moved by the particle in 12 seconds and also find the acceleration at *t=2.*

$t\ (Sec.)$: 0 2 4 6 8 10 12

$v(m/\sec.)$: 4 6 16 34 60 94 136

Ans: *$v=t^2-t+4$, Distance =552 m , Acc.=3 m/sec^2* **[Dec. 2002, Feb. 2010]**

2. Gregory- Newton's Backward difference interpolation formula:

[*When required value of y=f(x) is near to the bottom i.e. x_n , then use backward difference interpolation formulae. It is also used for extrapolating values of y for x , when x is slightly grater than x_n:*

$$y=f(x)=y_n+\frac{p}{1!}\nabla y_n+\frac{p(p+1)}{2!}\nabla^2 y_n+\ldots\ldots+\frac{p(p+1)\ldots\ldots(p+(p-1))}{n!}\nabla^n y_n\ldots\ldots\ldots\ldots+\left[Where\ p=\frac{(x-x_n)}{h}\right]$$

Q.24 From the following table , evaluate f(3.8) using Newton backward interpolation formulae:

x	0	1	2	3	4	**Ans:4321936**
y	1	1.50	2.20	3.10	4.60	

Q.25 The following table gives the values of a function , Find (i) $f(1.8)$ (ii) $f\,'(1.5)$ (iii) $\int_0^2 f(x)dx$

x	0	0.5	1	1.5	2
y	0.3989	0.3521	0.2420	0.1295	0.0540

Ans : $f(x)=0.3989+0.2105x-0.28782x^2+0.1242x^3-0.041433x^4$

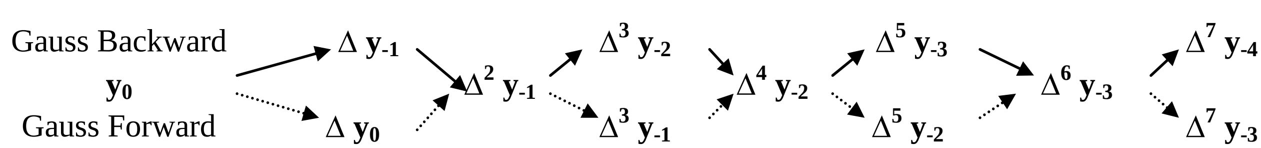
Central difference interpolation formulas: [When required value of y=f(x) is near to the middle , then use central difference interpolation formulae.]

1. Gauss forward difference interpolation formula:

$$y=f(x)=y_0+\frac{p}{1!}\Delta y_0+\frac{p(p-1)}{2!}\Delta^2 y_{-1}+\frac{(p+1)p(p-1)}{3!}\Delta^3 y_{-2}+\ldots\ldots\ldots(0<p<1)$$

2. Gauss Backward difference interpolation formula:

$$y=f(x)=y_0+\frac{p}{1!}\Delta y_{-1}+\frac{(p+1)p}{2!}\Delta^2 y_{-1}+\frac{(p+1)p(p-1)}{3!}\Delta^3 y_{-2}+\ldots\ldots\ldots(-1<p<0)$$



3. Sterling Formula: { Sterling formula is the mean of gauss forward and back ward formula}

$$y=f(x)=y_0+\frac{p}{1!}\left(\frac{\Delta y_0+\Delta y_{-1}}{2}\right)+\frac{p^2}{2!}\Delta^2 y_{-1}+\frac{(p+1)p(p-1)}{3!}\left(\frac{\Delta^3 y_{-1}+\Delta^3 y_{-2}}{2}\right)+\frac{p^2(p^2-1)}{4!}\Delta^4 y_{-2}\ldots,\ (\frac{-1}{4}<p<\frac{1}{4})$$

4. Bessel's Formula :{Shift the origin to 1 by replacing p by (p-1) & add 1 to each argument 0,-1,-2….in gauss backward formulas and , take mean of gauss forward formula and revised backward formula}

$$y = f(x) = y_0 + \frac{p}{1!}\Delta y_0 + \frac{p(p-1)}{2!}\left(\frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2}\right) + \frac{p(p-1)(p-1/2)}{3!}\Delta^3 y_{-1} + \frac{(p+1)p(p-1)(p-2)}{4!}\left(\frac{\Delta^4 y_{-2} + \Delta^4 y_{-1}}{2}\right) + \ldots,\ (\frac{1}{4} < p < \frac{3}{4})$$

Q.25 Use Stirling formulae to find y for x=35 from the following table:

x	20	30	40	50	**Ans: 395**
y	512	439	346	243	

Q.26 Given that $\sin 45^0=0.7071$, $\sin 50^0=0.7660$, $\sin 55^0=0.8192$, $\sin 60^0=0.8660$ find $\sin 52^0$ by using any method of interpolation. **Ans:0.7880**

Q.27 Use Stirling formula to evaluate f(1.22), given that

x	1	1.1	1.2	1.3
y	8.403	8.781	9.129	9.451

Ans: 9.19548 **[June 2007]**

Q.28 Employ Stirling formulae to compute $\boldsymbol{y_{12.2}}$ from the following table ($y_x=1+\log_{10}\sin x$)

x^0	10	11	12	13	14
$10^5 y_x$	23967	28060	31788	35200	38368

Ans:0.324937072 **[Dec. 2005]**

Interpolation with unequal intervals:

Divided difference: $f[x_0,x_1] = \dfrac{f(x_1)-f(x_0)}{x_1-x_0}$, $f[x_0,x_1] = \dfrac{f[x_1,x_2]-f[x_0,x_1]}{x_2-x_0}$

1. **Newton's Divided difference interpolation formula:**

$$f(x) = f(x_0) + (x-x_0)\, f[x_0,x_1] + (x-x_0)(x-x_1)\, f[x_0,x_1,x_2] + \ldots\ldots\ldots$$

1. **Lagrange's Interpolation formula:**

$$y = f(x) = \frac{(x-x_1)(x-x_2)\ldots\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots\ldots(x_0-x_n)} f(x_0) + \frac{(x-x_0)(x-x_2)\ldots\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots\ldots(x_1-x_n)} f(x_1) + \ldots\ldots\ldots\ldots$$

2. **Inverse Lagrange's Interpolation formula:**

$$x = f^{-1}(y) = \frac{(y-y_1)(y-y_2)\ldots\ldots(y-y_n)}{(y_0-y_1)(y_0-y_2)\ldots\ldots(y_0-y_n)}(x_0) + \frac{(y-y_0)(y-y_2)\ldots\ldots(y-y_n)}{(y_1-y_0)(y_1-y_2)\ldots\ldots(y_1-y_n)}(x_1) + \ldots\ldots\ldots\ldots$$

Q.26 Find a **polynomial** satisfied by (-4,1245) , (-1,33), (0,5), (2,9), and (5,1335) by Newton divided difference formula.**[Dec. 2015,June16]**

Q.27 For the given data find F(8) using (1) Lagrange's formula (2) Newton's divided difference formula

x:	4	5	7	10	11	13
$f(x)$:	48	100	294	900	1210	2028

ANS: 448 [RGPV. DEC. 2006]

Q.28 Use Newton's divided difference formula to find the form of *f(x)* given

x:	0	2	3	6
$f(x)$:	648	704	729	792

Ans: $f(x) = -x^2 + 30x + 648$ **[RGPV. DEC. 2008, JUNE 2006, JUNE 2015]**

Q.29 The following table is given . what is the form of **function**?

Q.30

x:	0	1	2	5
$f(x)$:	2	3	12	147

Ans: $2x^3 - 7x^2 + 6x + 1$**, y(4)=4 ,** **[RGPV: May 2019**

Q.31 Find the **polynomial** which takes the following values.

x:	1	3	5	7	9	11
$f(x)$:	3	14	19	21	23	28

, [RGPV: May 2019 Ex.]

Q.32 The values of *x* and *f*(*x*) are given below , Find the value of *f*(*x*) at x=4

x:	1	2	4	8
$f(x)$:	0	1	5	21

Q.33 From the following data , find f(9), using **Newton's divided difference**

x :	5	7	11	13	17
f(x):	150	392	1452	2366	5202

Ans: 810 , **[RGPV. June 2010, June 2014]**

Q.34 Given $\log_{10} 654=2.8156$, $\log_{10} 658=2.8182$, $\log_{10} 659=2.8189$, $\log_{10} 661=2.82.0$. Find by using **Newton divided difference formul**a the value of $\log_{10} 656$. **[May 2019 Ex.}**

Q.35 Apply **Lagrange's** method to find the values of x when y =15 from the Given data

$x:$	10	12	14	16	18	20
$f(x):$	2420	1942	1497	1109	790	540

Ans:1295 , **[RGPV. Dec. 2010]**

Q.36 Apply **Lagrange's** method to find the values of x when y =13 from the Given data

x:	30	35	40	45
f(x):	15	14	17	16

Q.37 Apply **Lagrange's** method to find the values of *x* when *y* =15 from the Given data **[June 16]**

x:	5	6	9	11
f(x):	12	13	14	16